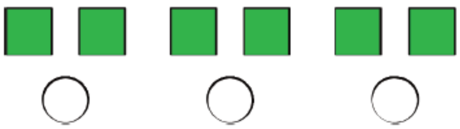
Unit 2 Ratio Lesson Summaries

Lesson Summaries are found at the end of the student version of every lesson. I have combined all of the Lesson Summaries into one document for ease of reviewing.

Lesson 1 Summary

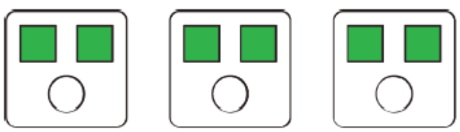
A **ratio** is an association between two or more quantities. There are many ways to describe a situation in terms of ratios. For example, look at this collection:



Here are some correct ways to describe the collection:

- The ratio of squares to circles is $6 : 3$.
- The ratio of circles to squares is 3 to 6.

Notice that the shapes can be arranged in equal groups, which allow us to describe the shapes using other numbers.



- There are 2 squares for every 1 circle.
- There is 1 circle for every 2 squares.

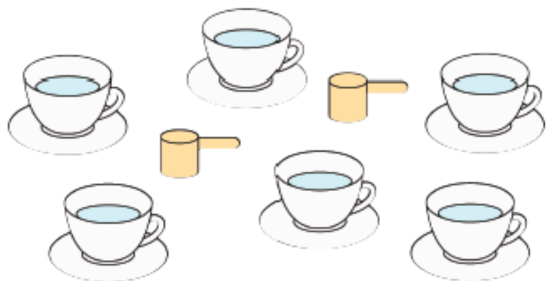
Lesson 1 Glossary Terms

- ratio

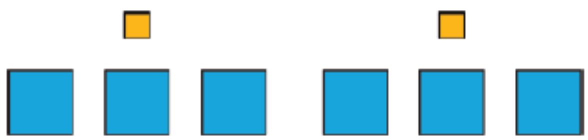
Lesson 2 Summary

Ratios can be represented using diagrams. The diagrams do not need to include realistic details. For example, a recipe for lemonade says, “Mix 2 scoops of lemonade powder with 6 cups of water.”

Instead of this:



We can draw something like this:



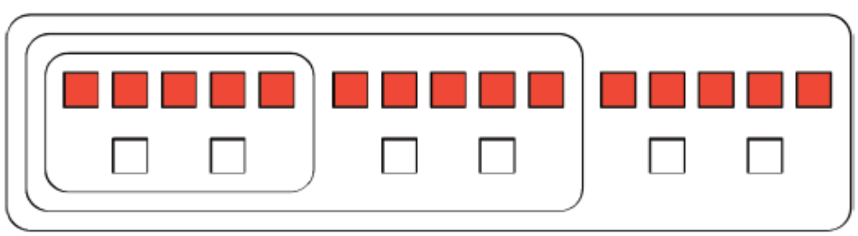
This diagram shows that the ratio of cups of water to scoops of lemonade powder is 6 to 2. We can also see that for every scoop of lemonade powder, there are 3 cups of water.

Lesson 3 Summary

A recipe for fizzy juice says, “Mix 5 cups of cranberry juice with 2 cups of soda water.”

To double this recipe, we would use 10 cups of cranberry juice with 4 cups of soda water. To triple this recipe, we would use 15 cups of cranberry juice with 6 cups of soda water.

This diagram shows a single batch of the recipe, a double batch, and a triple batch:



We say that the ratios $5 : 2$, $10 : 4$, and $15 : 6$ are **equivalent**. Even though the amounts of each ingredient within a single, double, or triple batch are not the same, they would make fizzy juice that tastes the same.

Lesson 3 Glossary Terms

- equivalent ratios

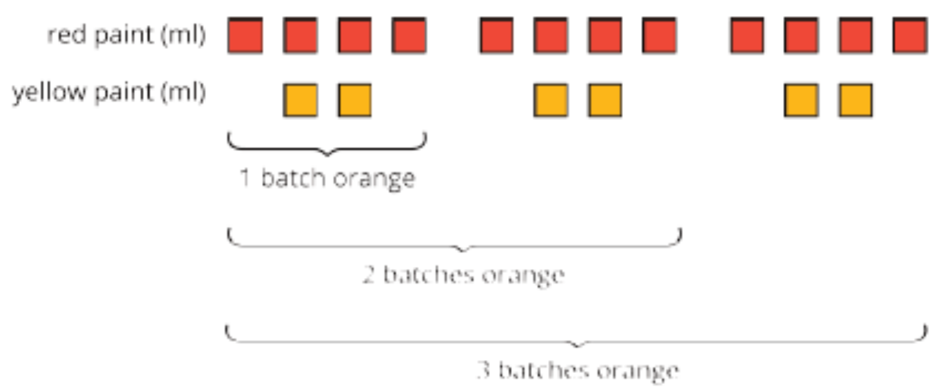
Lesson 4 Summary

When mixing colors, doubling or tripling the amount of each color will create the same shade of the mixed color. In fact, you can always multiply the amount of *each* color by *the same number* to create a different amount of the same mixed color.

For example, a batch of dark orange paint uses 4 ml of red paint and 2 ml of yellow paint.

- To make two batches of dark orange paint, we can mix 8 ml of red paint with 4 ml of yellow paint.
- To make three batches of dark orange paint, we can mix 12 ml of red paint with 6 ml of yellow paint.

Here is a diagram that represents 1, 2, and 3 batches of this recipe.

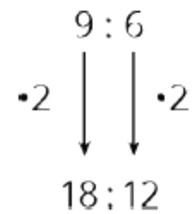


We say that the ratios $4 : 2$, $8 : 4$, and $12 : 6$ are *equivalent* because they describe the same color mixture in different numbers of batches, and they make the same shade of orange.

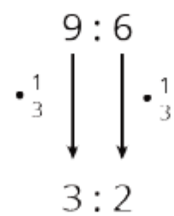
Lesson 5 Summary

All ratios that are **equivalent** to $a : b$ can be made by multiplying both a and b by the same number.

For example, the ratio $18 : 12$ is equivalent to $9 : 6$ because both 9 and 6 are multiplied by the same number: 2.

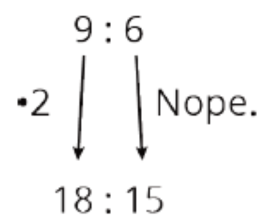


$3 : 2$ is also equivalent to $9 : 6$, because both 9 and 6 are multiplied by the same number: $\frac{1}{3}$.



Is $18 : 15$ equivalent to $9 : 6$?

No, because 18 is $9 \cdot 2$, but 15 is *not* $6 \cdot 2$.

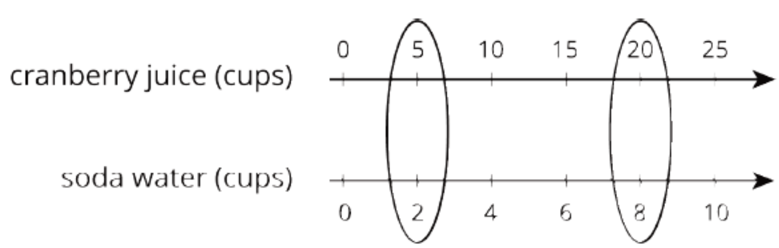


Lesson 5 Glossary Terms

- equivalent ratios

Lesson 6 Summary

You can use a **double number line diagram** to find many equivalent ratios. For example, a recipe for fizzy juice says, “Mix 5 cups of cranberry juice with 2 cups of soda water.” The ratio of cranberry juice to soda water is $5 : 2$. Multiplying both ingredients by the same number creates equivalent ratios.



This double number line shows that the ratio $20 : 8$ is equivalent to $5 : 2$. If you mix 20 cups of cranberry juice with 8 cups of soda water, it makes 4 times as much fizzy juice that tastes the same as the original recipe.

Lesson 6 Glossary Terms

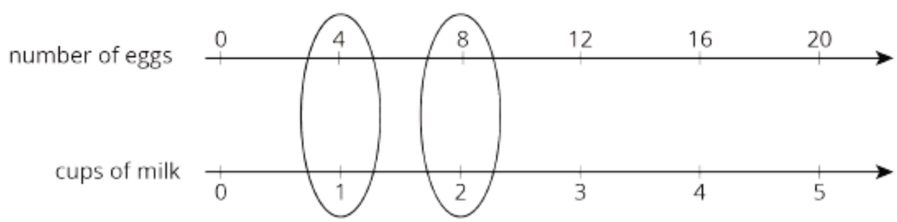
- double number line diagram

Lesson 7 Summary

Here are some guidelines to keep in mind when drawing a double number line diagram:

- The two parallel lines should have labels that describe what the numbers represent.
- The tick marks and numbers should be spaced at equal intervals.
- Numbers that line up vertically make equivalent ratios.

For example, the ratio of the number of eggs to cups of milk in a recipe is $4 : 1$. Here is a double number line that represents the situation:



We can also say that this recipe uses “4 eggs per cup of milk” because the word **per** means “for each.”

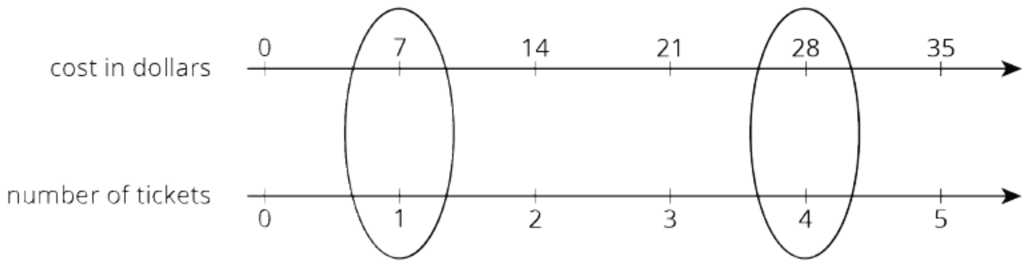
Lesson 7 Glossary Terms

- per

Lesson 8 Summary

The **unit price** is the price of 1 thing—for example, the price of 1 ticket, 1 slice of pizza, or 1 kilogram of peaches.

If 4 movie tickets cost $28, then the unit price would be the cost *per* ticket. We can create a double number line to find the unit price.



This double number line shows that the cost for 1 ticket is $7. We can also find the unit price by dividing, $28 \div 4 = 7$, or by multiplying, $28 \cdot \frac{1}{4} = 7$.

Lesson 8 Glossary Terms

- unit price

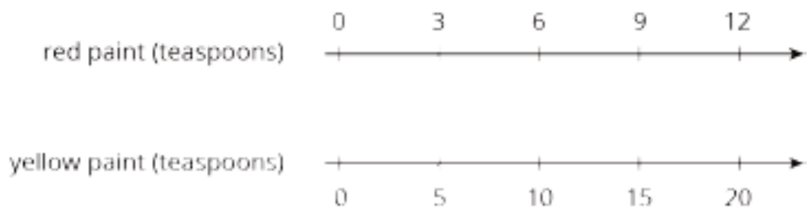
Lesson 10 Summary

Sometimes we want to know whether two situations are described by the **same rate.** To do that, we can write an equivalent ratio for one or both situations so that one part of their ratios has the same value. Then we can compare the other part of the ratios.

For example, do these two paint mixtures make the same shade of orange?

- Kiran mixes 9 teaspoons of red paint with 15 teaspoons of yellow paint.
- Tyler mixes 7 teaspoons of red paint with 10 teaspoons of yellow paint.

Here is a double number line that represents Kiran's paint mixture. The ratio $9 : 15$ is equivalent to the ratios $3 : 5$ and $6 : 10$.



For 10 teaspoons of yellow paint, Kiran would mix in 6 teaspoons of red paint. This is less red paint than Tyler mixes with 10 teaspoons of yellow paint. The ratios $6 : 10$ and $7 : 10$ are not equivalent, so these two paint mixtures would not be the same shade of orange.

When we talk about two things happening at the same rate, we mean that the ratios of the quantities in the two situations are equivalent. There is also something specific about the situation that is the same.

- If two ladybugs are moving at the same rate, then they are traveling at the *same constant speed*.

- If two bags of apples are selling for the same rate, then they have the *same unit price*.

- If we mix two kinds of juice at the same rate, then the mixtures have the *same taste*.

Lesson 12 Summary

Finding a row containing a “1” is often a good way to work with tables of equivalent ratios. For example, the price for 4 lbs of granola is \$5. At that rate, what would be the price for 62 lbs of granola?

Here are tables showing two different approaches to solving this problem. Both of these approaches are correct. However, one approach is more efficient.

- Less efficient

	granola (lbs)	price (\$)	
	4	5	
•2	8	10	•2
•2	16	20	•2
•2	32	40	•2
•2	64	80	•2
- 2 lbs	62	77.50	- \$2.50

- More efficient

	granola (lbs)	price (\$)	
	4	5	
$\cdot \frac{1}{4}$	1	1.25	$\cdot \frac{1}{4}$
•62	62	77.50	•62

Notice how the more efficient approach starts by finding the price for 1 lb of granola.

Remember that dividing by a whole number is the same as multiplying by a unit fraction. In this example, we can divide by 4 or multiply by $\frac{1}{4}$ to find the unit price.

Lesson 14 Summary

To solve problems about something happening at the same rate, we often need:

- Two pieces of information that allow us to write a ratio that describes the situation.

- A third piece of information that gives us one number of an equivalent ratio. Solving the problem often involves finding the other number in the equivalent ratio.

Suppose we are making a large batch of fizzy juice and the recipe says, “Mix 5 cups of cranberry juice with 2 cups of soda water.” We know that the ratio of cranberry juice to soda water is $5 : 2$, and that we need 2.5 cups of cranberry juice per cup of soda water.

We still need to know something about the size of the large batch. If we use 16 cups of soda water, what number goes with 16 to make a ratio that is equivalent to $5 : 2$?

To make this large batch taste the same as the original recipe, we would need to use 40 cups of cranberry juice.

cranberry juice (cups)	soda water (cups)
5	2
2.5	1
40	16

Lesson 15 Summary

A **tape diagram** is another way to represent a ratio. All the parts of the diagram that are the same size have the same value.

For example, this tape diagram represents the ratio of ducks to swans in a pond, which is $4 : 5$.

The first tape represents the number of ducks. It has 4 parts.

The second tape represents the number of swans. It has 5 parts.

There are 9 parts in all, because $4 + 5 = 9$.

Suppose we know there are 18 of these birds in the pond, and we want to know how many are ducks.

The 9 equal parts on the diagram need to represent 18 birds in all. This means that each part of the tape diagram represents 2 birds, because $18 \div 9 = 2$.

There are 4 parts of the tape representing ducks, and $4 \cdot 2 = 8$, so there are 8 ducks in the pond.

Lesson 15 Glossary Terms

- tape diagram

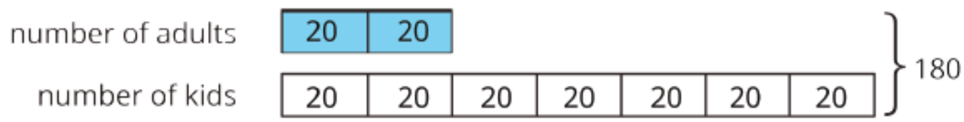
Lesson 16 Summary

When solving a problem involving equivalent ratios, it is often helpful to use a diagram. Any diagram is fine as long as it correctly shows the mathematics and you can explain it.

Let's compare three different ways to solve the same problem: The ratio of adults to kids in a school is $2 : 7$. If there is a total of 180 people, how many of them are adults?

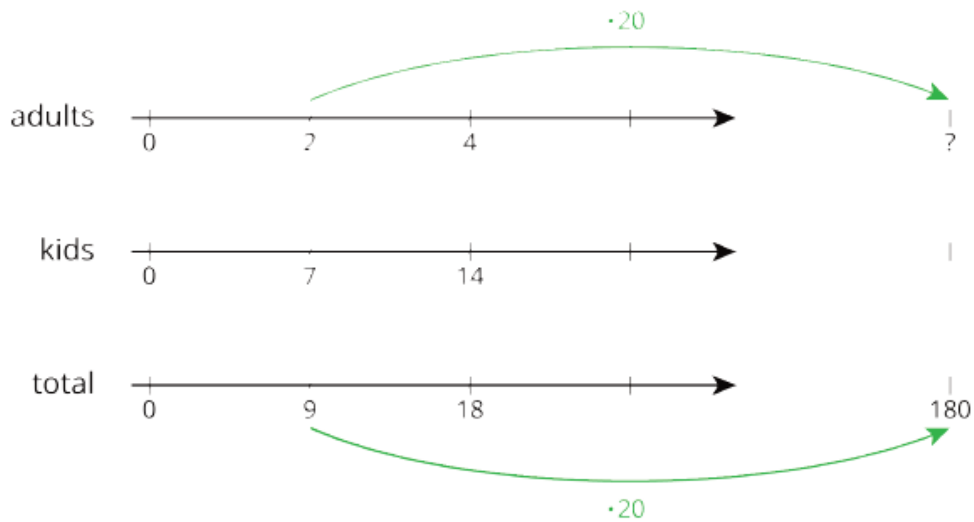
- *Tape diagrams* are especially useful for this type of problem because both parts of the ratio have the same units ("number of people") and we can see the total number of parts.

This tape diagram has 9 equal parts, and they need to represent 180 people total. That means each part represents $180 \div 9$, or 20 people.



Two parts of the tape diagram represent adults. There are 40 adults in the school because $2 \cdot 20 = 40$.

- *Double or triple number lines* are useful when we want to see how far apart the numbers are from one another. They are harder to use with very big or very small numbers, but they could support our reasoning.



- *Tables* are especially useful when the problem has very large or very small numbers.

adults	kids	total
2	7	9
?		180

(·20 down both sides)

We ask ourselves, “9 times what is 180?” The answer is 20. Next, we multiply 2 by 20 to get the total number of adults in the school.